*Invention Journal of Research Technology in Engineering & Management (IJRTEM)*
*ISSN: 2455-3689*
*www.ijrtem.com Volume 2 Issue 3 ǁ March. 2018 ǁ PP 07-12*

# Advanced Stability Analysis of Control Systems with Variable Parameters

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***ABSTRACT:*** *The purpose of the current research is to advance further the D-Partitioning method and emphasize on its practical application. It has the objective to clarify it in a user friendly manner in order to simplify its implementation. By applying the basic initial ideas of the method, the main line of the research is the development of a generalized stability analysis tool and demonstrating its application. With the aid of this tool, proper parameter values can be chosen for a desirable performance and stability of a system. The analysis tool can be practically used when one, two or more system's parameters are varied independently or simultaneously. Basically this tool defines regions of stability in the space of the system's parameters.*

***KEYWORDS:*** *Advanced D-Partitioning, Stability Analysis Tool, Variable Parameters, Marginal Cases*

## I. INTRODUCTION

By applying the basic initial ideas of Neimark [1], the aim of this research is to achieve further advancement of the D-Partitioning method, developing a general stability analysis tool and demonstrating its application. By implementing an interactive MATLAB procedure methodology, the advancement of the D-Partitioning represented in this research is based on the innovative transparent graphical display of the simultaneous interaction between n-variable parameters, as well as the determination of the simultaneous marginal values of n-variable parameters. This innovative analysis tool is classified as the method of Advanced D-Partitioning.

The advancement of the analysis developed in this research is in terms of its practical application in case one or two system's parameters that are varied independently or simultaneously. It is a further upgrade of the author's previous work [2], [3]. The stability analysis is based on the fact that the roots of the characteristic equation of a control system depend on the coefficients of this equation and therefore depend on the system's parameters [4]. Generally, for an *n*-order characteristic equation, *m* roots may be positioned in the right-hand side and (*n* - *m*) in the left-hand side of the s-plane, being a reason for the developing of this original method of stability analysis. The application of the suggested method of Advanced D-Partitioning is demonstrated in case of real-life control systems.

## II. ADVANCED D-PARTITIONING IN CASE OF ONE VARIABLE PARAMETER

To implement the method of the Advanced D-partitioning, the *n*-order characteristic equation of a control system is presented in the format:

$$G(s) = a_0 s^n + a_1 s^{n-1} + \ldots + a_n = 0 \tag{1}$$

where $s$ is the Laplace operator

$a_0, a_1, \ldots, a_n$ - are parameter-dependent coefficients

Initially, the hypothetical system characteristic equation (1) is introduced in the following format to expose a variable parameter:

$$G(s) = P(s) + vQ(s) = 0 \tag{2}$$

where $P(s)$ and $Q(s)$ are polynomials of the Laplace operator $s$
$\quad \nu$ is the single variable system parameter

The borders of the D-partitioning can be determined by substituting $s = j\omega$ in the characteristic equation (2) and varying the frequency within the range $-\infty \leq \omega \leq +\infty$. Accordingly, the equation (2) is transformed to equation (3) as follows:

$$G(j\omega) = P(j\omega) + \nu Q(j\omega) = 0$$

(3)

Further, ν can be presented also as a complex number as follows:

$$\nu = -\frac{P(j\omega)}{Q(j\omega)} = X(\omega) + jY(\omega)$$

(4)

Real part $X(\omega)$ of this complex number corresponds in reality to the value of the variable parameter of the control system. Then, the D-partitioning regions can be obtained graphically in the complex plane $\nu = X(\omega) + Y(\omega)$, by varying the frequency within the range $-\infty \leq \omega \leq +\infty$.

In this research, a number of original examples of practical implementation of the method will be demonstrated. As initial example, a real-life **cruise control system** is reduced to a third order system of Type 0 [5], [6]. Cruise control is a system that automatically controls the speed of a motor vehicle. The open loop transfer function of the system is with a variable parameter that is its **gain factor *K*** and is represented as follows:

$$G_O(s) = \frac{1000K}{(s+10)(s+50)(s+100)} = \frac{1000K}{s^3 + 160s^2 + 6500s + 50000}$$

(5)

The transfer function of the feedback system can be represented as:

$$G_{CL}(s) = \frac{Go(s)}{1 + Go(s)} = \frac{1000K}{(s+10)(s+50)(s+100) + 1000K}$$

(6)

It is suggested that the original **gain factor *K*** of the system is a variable parameter due to some temperature effects within the environment of its operation. The characteristic equation of the feedback system is:

$$G(s) = (s+10)(s+50)(s+100) + 1000K = 0$$

(7)

To expose the variable parameter equation (7) is presented as:

$$G(s) = P(s) + KQ(s) = 0$$

(8)

Where the polynomials of equation (9) are as follows:

$$P(s) = (s+10)(s+50)(s+100)$$

(9)

$$Q(s) = 1000$$

(10)

Then, the variable parameter is presented as:

$$K(s) = -\frac{P(s)}{Q(s)} = -\frac{(s+10)(s+50)(s+100)}{1000} = -\frac{s^3 + 160s^2 + 6500s + 50000}{1000}$$
(11)

**The D-partitioning curve in terms of one variable parameter can be plotted in the complex plane within the frequency range –∞ ≤ ω ≤ +∞, facilitated by MATLAB the "Nyquist" m-code,** this is illustrated for the case of the variable gain factor *K* in Fig. 1.

```
>> K=tf([1 160 6500 50000],[1000])
>> nyquist(K)
```

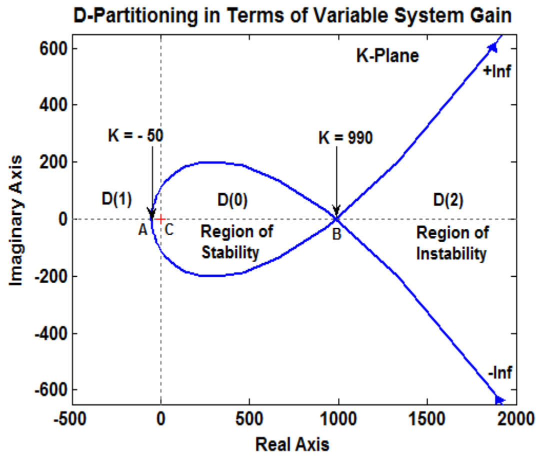


Figure 1: *D-Partitioning Facilitated by the "nyquist" m-code*

To avoid any misunderstanding or misinterpretation of the D-Partitioning procedure, the "nyquist" m-code can be modified into a "dpartition" m-code with the aid of the MATLAB Editor and a proper formatting. It is seen that the D-partitioning determines three regions on the *K*-plane: D(0), D(1) and D(2). **Only D(0) is the region of stability, being the one, always on the left-hand side of the curve for a frequency variation from –∞ to +∞. The system gain is 0.02*K***, while ***K* is considered as a gain factor**.

The results, obtained from the D-partitioning are also compared and confirmed with the outcome from the Nyquist analysis for the cases of the positive gain factors $K \in [400,1200]$ corresponding to gains $0.02K \in [8,24]$ accordingly, as shown in Fig. 2. To examine the effect of the variable gain factor *K* on the system's stability, an LTI array of *Ao* is created.

```
>> K=[400:50:1200];
>> for n=1:length(K)
Ao_array(:,:,n)=tf([1000*K(n)], [1 160 6500 50000]);
end
>> Ao1=tf([990000], [1 160 6500 50000])
nyquist(Ao_array,Ao1)
>> nyquist(Ao_array,Ao1)
```

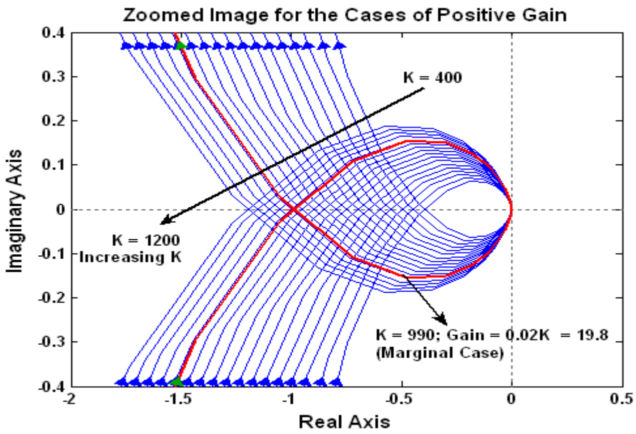


Figure 2: *Zoomed Image of the Marginal Gain, Confirmed with the Aid of the Nyquist Stability Criterion*

The case of $\boldsymbol{K}$ **= 990, or a system gain 0.02*K* = 19.8 corresponds to a marginal case**, while $K$ = 1200, corresponding to $0.02K$ = 24 corresponds again to an unstable state of the system. In this case the negative values of $\boldsymbol{K}$ are ignored. The results from the Nyquist analysis prove the outcome of the Advanced D-Partitioning and confirm that the system will be stable if $K < 990$, **corresponding system gain 0.02*K* < 19.8.**

## III. ADVANCED D-PARTITIONING IN CASE OF TWO VARIABLE PARAMETERS

If two of the system parameters are variable simultaneously [2], [3], [5] the system general characteristic equation (1) can be presented as:

$$G(s) = \mu P(s) + \gamma Q(s) + R(s) = 0$$

(12)

where *P(s)*, *Q(s)*, and *R(s)* are polynomials of the Laplace operator $s$
$\mu$ and $\gamma$ are variables equal to the system's variable parameters

The analysis can be demonstrated for the control system of an **armature-controlled dc motor and a type-driving mechanism** [7], [8]. In this case the gain and one of the time-constants are uncertain and variable. The open-loop transfer function of the system is presented as:

$$G(s) = \frac{K}{(1+Ts)(1+0.5s)(1+0.8s)}$$

(13)

By substituting $s = j\omega$ equation, the characteristic equation of the unity feedback system is:

$$K + (1+Ts)(1+0.5s)(1+0.8s) = -1 + (1.3T + 0.4)\omega^2 + j\omega(0.4T\omega^2 - 1.3 - T) = 0$$

(14)

Since the gain may obtain only real values, the imaginary term of equation (14) is set to zero, from where:

$$\omega^2 = \frac{1.3 + T}{0.4T}$$

(15)

The result of (15) is substituted into the real part of equation (14), from where:

$$K = \frac{1.3T^2 + 1.69T + 0.52}{0.4T} = 3.25T + 4.225 + \frac{1.3}{T}$$
(16)

The D-Partitioning curve $K = f\ (T)$ is plotted with the aid of the following code:

```
>> T = 0:0.1:5;
>> K = 3.25.*T+4.225+1.3./T
  K =
  Columns 1 through 10
  Inf   17.5500  11.3750   9.5333   8.7750   8.4500   8.3417   8.3571   8.4500   8.5944
  Columns 11 through 20
  8.7750   8.9818   9.2083   9.4500   9.7036   9.9667  10.2375  10.5147  10.7972  11.0842
  Columns 21 through 30
  11.3750  11.6690  11.9659  12.2652  12.5667  12.8700  13.1750  13.4815  13.7893  14.0983
  Columns 31 through 40
  14.4083  14.7194  15.0313  15.3439  15.6574  15.9714  16.2861  16.6014  16.9171  17.2333
  Columns 41 through 50
  17.5500  17.8671  18.1845  18.5023  18.8205  19.1389  19.4576  19.7766  20.0958  20.4153
  Column 51
  20.7350
>> plot(T,K)
```

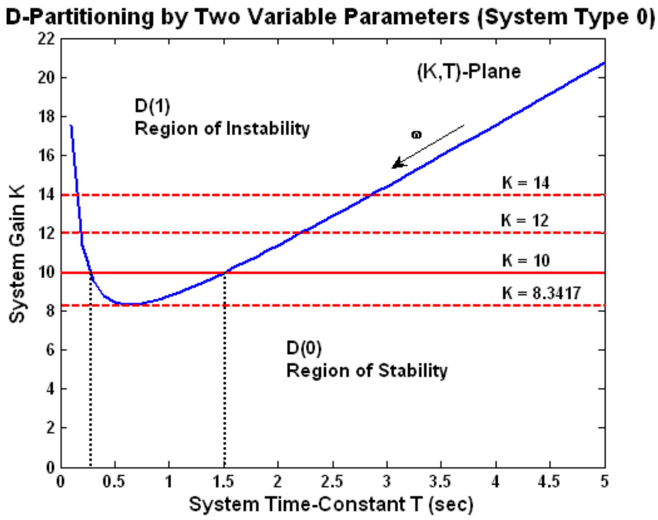


Figure 3: *D-Partitioning in terms of Two Variable Parameters*

The D-Partitioning curve $K = f\ (T)$ defines the border between the region of stability D(0) and instability D(1) for the case of simultaneous variation of the two system parameters. Each point of the D-Partitioning curve represents also the marginal values of the two simultaneously variable parameters. This is a unique advancement and an novelty in the theory of control systems stability analysis.

The system performance in case of variation of the time-constant $T$ is examined at gain set to $K$ = 10. **When 0 < $T$ < 0.25 sec and $T$ > 1.5 sec the system is stable.** But it becomes **unstable in the range 0.25 sec < $T$ < 1.5 sec.** The system performance can also be investigated for any other values of the variable gain $K$ ($K$ = 12, 14, etc.). It is obvious that if $K$ is varied, this affects the values of $T$ at which the system may become unstable. Higher values of $K$, enlarges the range of $T$ at which the system will fall into instability.

If $K$ < 8.3417, a limit determined with the aid of MATLAB interface procedure, the system is stable for any value of the $T$. It is obvious that the system performance and stability depends on the interaction between the two simultaneously varying parameters.

## III. CONCLUSIONS

By applying the Advanced D-partitioning in case of one variable system parameter, the D-Partitioning curve, plotted on the complex plane of the variable parameter, develops regions of stability and instability and showing a clear picture of the parameter limits of variation to keep the system stable. The Advanced D-partitioning in case two variable parameters is demonstrating the strong interaction between the variable parameters. Also, each point of the D-Partitioning curve represents the marginal values of the two simultaneously variable parameters. This is a unique phenomenon in the advancement and is an innovation in the theory of control systems stability analysis.

The Advanced D-partitioning analysis in case of one and two variable parameters is considerably more convenient and quicker method for stability analysis, compared with any other method. This research contributes to knowledge, since there is no other known stability analysis method in control theory that can illustrate graphically the system's regions of stability and instability in case of multivariable parameters and to demonstrate the interaction and the simultaneous marginal values of a number of variable parameters. Only the developed Advanced D-Partitioning analysis has these unique properties.

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